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Empirical Models, Rules, and Optimization: Turning Positive Economics on its Head

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Abstract

This paper considers supply decisions by firms in a dynamic setting with adjustment costs and compares the behavior of an optimal control model to that of a rule-based system which relaxes the assumption that agents are explicit optimizers. In our approach, the economic agent uses believably simple rules in coping with complex situations. We estimate rules using an artificially generated sample obtained by running repeated simulations of a dynamic optimal control model of a firm's hiring/firing decisions. We show that (i) agents using heuristics can behave *as if* they were seeking rationally to maximize their dynamic returns; (ii) the approach requires fewer behavioral assumptions relative to dynamic optimization and the assumptions made are based on economically intuitive theoretical results linking rule adoption to uncertainty; (iii) the approach delineates the domain of applicability of maximization hypotheses and describes the behavior of agents in situations of economic disequilibrium.

The approach adopted uses concepts from fuzzy control theory. An agent, instead of optimizing, follows Fuzzy Associative Memory (FAM) rules which, given input and output data, can be estimated and used to approximate any non-linear dynamic process. Empirical results indicate that the fuzzy rule-based system performs extremely well in approximating optimal dynamic behavior in situations with limited noise. Simulations are also performed under increasingly noisy.

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Introduction

The representation of agent behavior is a central issue for any empirical economic modeling and much economic theory. The diversity of approaches in the literature shows that, even though neoclassical "as if" maximization dominates much of the discussions, the issue is far from resolved. This paper considers firms' decisions in a dynamic setting and compares the behavior of a rule-based system relative to an explicit optimal control model. In the process, we relax the assumption that agents are optimizers. In our approach, the economic agent will use believably simple rules in coping with complex situations and, while allowing by "profit-seeking" motives to shape its behavior rules, will not be explicitly searching for a model's first order conditions. The approach adopted uses concepts and methods from fuzzy control theory.

We compare the empirical performance of a rule-based system relative to a benchmark represented by the optimal control solution of a firm's hiring/firing decisions when facing fluctuating product prices and significant adjustment costs starting from an analytic model by Dixit (1997). The problem can be approached in two ways: the first is to assume the rules to be known based on expert knowledge (in this case of firm managers), while the second is to estimate the rules assuming no prior knowledge. In this paper we follow the estimation approach because we are interested in three issues: (i) can behavioral rules be estimated which replicate "real world" economic behavior; (ii) if rules are estimated, can such rules perform "close enough to optimality" to support the positivist economics approach of "as if" optimization (Friedman, 1953); (iii) if so, are there situations where the rules-based system breaks down and deviates substantially from optimality?

Our results indicate that, while we are able to replicate optimizing behavior when agents face normal circumstances, the model's outcome may interestingly deviate from the behavior of standard neoclassical economic models when the complexity of the environment increases relative to what agents routinely handle. Equally important, we show that it is possible to estimate behavior rules from data rather than either assume them or try to obtain them by simulating the interactions of adaptive agents in a microsimulation framework.

Simple Rules and Complex Behavior

We start from an assumption that systemic behavior emerges from the interaction and coordination of many agents who learn from experience with limited information. This emergent behavior may, or may not, coincide with the positivist view that agents behave "as if" optimizing. Our approach is that agents are faced with information as input and then make decisions using economic reasoning. We assume that rules can approximate such processes.

An elegant theoretical justification for the existence and stability of such rules is provided by Heiner (1983 and 1989), who formally characterizes why rule-based behavior appears to be associated with a variety of situations faced by agents. Administrative procedures, rules of thumb, and social norms all express the tendency of agents to restrict their set of possible actions to ensure successful outcomes. Heiner (1983) defines a *reliability ratio* for rules, and states: "…when to allow flexibility to select an additional action: do so if the actual reliability in selecting the action exceeds the minimum required reliability necessary to improve performance." The implication of his findings is that an agent should ignore rules that are appropriate only in rare or unusual situations.[1] In general, rules will not cover the whole repertoire of available actions an agent could undertake. An agent need only be capable of determining when to select particular actions from a limited range of allowable alternatives.

Alchian (1950), in his classic paper "Uncertainty, Evolution, and Economic Theory", discusses in detail the relationship between observed behavior and uncertainty:

> Observable patterns of behavior and organization are predictable in terms of their relative probabilities of success or viability *if* they are tried. The observed prevalence of a type of behavior depends upon both this probability of viability and the probability of the different types being submitted to the economic system for testing and selecting.

And on the relation between economic analysis and agent behavior:

[1] The approach adopted by Heiner (1983) refers to a single agent but is not limited to an individual. All that is required to adopt the approach is an objective (which may be an institutional or economy-wide objective) and knowledge about uncertainty; in fact, the author applies the approach to a diverse set of issues at different levels of aggregation.

> … the economist can diagnose the particular attributes which were critical in facilitating survival [of a firm], even though the individual participants were not aware of them.

He concludes that the existence of uncertainty is the basis of evolution in that it is driven by adaptive imitative behavior.[2] Alchian's approach is exceptional because it does not base its aggregate description on individual optimal action; yet it does not destroy the basis of prediction, explanation, or diagnosis.[3] Heiner (1983) also argues that, to determine when to select a particular action, "the agent does not require an ability to understand why the resulting behavior patterns evolved" and highlights the importance of (i) the probability of the right circumstances occurring for an action, (ii) the viability of an action (probability of "correctly" responding to circumstances), and (iii) implicit in his argument, that an action be tried (to determine the reliability of such action).

Rules describe the adaptation of individuals, institutions, or the economy as a whole (depending on the level of aggregation of an analysis) only to situations which are relatively likely. The plausibility of modeling agent behavior as rule-based and not as consciously maximizing is very appealing in relation to the current literature on cognition. Moreover, from the point of view of economics, it allows us to bypass the controversy concerning the validity of assuming representative agents. Using the framework proposed here for the firm, analysis at a more aggregate level need not assume away heterogeneity for the framework to be internally consistent. As Kirman (1992) points out in his critique of the representative agent (as it is currently used), many interacting heterogeneous agents could give rise to very regular characteristics in the aggregate. This implies that, if the regularities are captured through rules estimated from "normal" data, these rules may apply only in a neighborhood of normality. Even in situations where an abrupt shock occurs, in which case the rules may have

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[2] Tintner (1941) convincingly stated that profit maximization made no sense when agents have imperfect foresight and limited abilities to solve complex problems. His view that agents would make a decision whose potential outcome *distribution* is preferable is exactly true in Heiner's sense. In the bounded rationality literature, Alchian (1950) is associated with the profit-maximizing hypothesis. In the article, however, he agrees with Tintner. In fact, Alchian goes even further and says that, even at the economic system level, the profit-maximization hypothesis is invalid because "realized profits, not maximum profits, are the mark of success and viability".

[3] One point of departure from Alchian's paper was the development of stochastic dynamic models (Chow, 1975); however, the conditionality on actions having been tried is completely lost in the stochastic optimization approach.

to be revised, it may be interesting to see how normal rules perform under shock (whether, in fact, they do replicate actual behavior).[4] In terms of disequilibrium dynamics, following a shock, a rule-based model offers more predictive capacity than a neoclassical equilibrium model which, by construction, has nothing to contribute in disequilibrium situations.

We develop a framework that can retain the usefulness of the "representative agent" approach because we are interested in a method that can be applied to real-world policy questions using empirical models. We realize the importance of interaction between agents in determining aggregate outcomes; however, we are also aware that trying to obtain real-world scenarios by simulating all the interactions among agents in an economy is a very arduous task. The development of such an agent-based micro-simulation approach is difficult given the likely existence of path-dependent multiple outcomes.[5] In addition, we assume that systemic behavioral rules can be estimated from data. The "representative agent" in our context thus embodies the rules associated with a "problem class" incorporating agent behavior at various levels of aggregation, without resorting to explicit optimization behavioral assumptions. We do not aim to solve the problems faced by agent-based micro-simulation methods; rather, we circumvent them by allowing the simple rules to be associated with aggregate behavior.

Our underlying objective is to provide an applied framework for recognizing when an optimization framework is appropriate, and if not appropriate, provide an alternative tool for modeling economic behavior. The problem can be separated into three components: a learning method, a mapping between a system's inputs and outputs obtained using the learning method, and a test of how the mapping can replicate the behavior of the system. In this paper, while addressing all three components, we focus on how to represent the input-output mapping in a way that can be interpreted in economic terms and how it performs in

[4] This would be a step in addressing the problem referred to by Leijonhufvud (1981) when he characterizes general equilibrium theory as modeling systems that always work well and Keynesian theory as modeling systems that never work.

[5] At an intuitive level, individual agents can trigger historical events that modify the state of society (viewed as a non-linear dynamic system). While such events are rare, and hardly in the domain of economic analysis, any system of micro-simulation of agents that deviates from the optimization approach of neoclassical economics and aspires to describe real-world events, will have to deal with the ramifications of "history". If multiple equilibria create problems (from a policy analyst's perspective) in restrictive, ahistoric settings like game theory and general equilibrium theory, such problems are unlikely to be solved by agent-based simulations.

tracking the true "data generating" process.[6] The learning method is very simple and does not presume any mathematical sophistication on the part of agents.

A Rule-Based Model of Dynamic Processes

Friedman (1953) argues that the only relevant test of the validity of a hypothesis is comparison of its predictions with experience. Interestingly, Friedman continues by stating that:

> … among alternative hypothesis equally consistent with the available evidence … relevant considerations are suggested by the criteria of "simplicity" and "fruitfulness". A theory is "simpler" the less the initial knowledge to make predictions … [and] it is more "fruitful" the more precise the resulting prediction, the wider the area within which the theory yields predictions, and the more additional lines for further research it suggests.

In this paper we perform a "thought experiment" of the following kind: we first show that agents (firms) using behavior rules, or heuristics, can perform tasks *as if* they were seeking rationally to maximize their returns (discounted profits). This is done by using an artificially generated sample obtained by running repeated simulations of an optimal control representation of a firm's dynamic hiring/firing decisions. Having established that the *rule-based* and the *as-if* hypotheses are equally consistent with the data (the latter is valid because of how the sample is generated), we then continue by demonstrating that the behavior rule approach is "simple" and "fruitful".

The ongoing debate about modeling the behavior of economic agents can be roughly represented as lying between two extremes. On one side the dominant strand of literature relies on "representative agent" models in which the behavior of a system is determined by the solution of an optimization problem for an "average" agent in the model. On the other hand, the new literature on "artificial adaptive agents" represents the economy as a network of interacting agents having limited ability to process information (Leijonhufvud, 1993). In between these two extremes there is the rapidly evolving literature on bounded rationality:

[6] The economic interpretability is an important component in a modeling framework and one that is lacking in other techniques that have been imported from artificial intelligence applications such as neural network mappings.

(i) **“satisficing”** where agents perform limited searches accepting the first satisfactory decision (Simon, 1955; Day and Tinney, 1968; Winter 1971)

(ii) **suboptimization**: an optimizing agent facing a difficult decision solves a simpler approximate optimization problem rather than the formally correct, but more difficult problem (Day, 1963; Day and Cigno, 1978)

(iii) **heuristics**: adoption of rules of thumb; in economics references are Winter (1982), Akerlof and Yellen (1987), Thaler (1988), Arthur (1994).

(iv) **institutional processes**: the existence and behavior of economic institutions depends on the need to economize on transaction costs (Williamson, 1986)

(v) **evolution of markets**: survival of “as if” firms (Winter,1971; Conlisk, 1983)

(vi) **classifier systems**: agents choose among a discrete list of actions and they learn through trial-and-error, associating probability weights to actions based on historical rewards (Holland and Miller, 1991)

(vii) **deliberation costs**: model the trade-off between effort devoted to deliberation and that devoted to other activities.

Conlisk (1996) provides an in-depth review of the different treatments of bounded rationality. The method to be presented here relaxes the assumption that agents are optimizers without precluding the artifice of a representative agent and falls under the rubric of “heuristics”. However, it can also be viewed as a model-free approximation of the agent-environment interaction in a dynamic system with underlying unknown equations of motion.

It is widely accepted that economic agents adapt to their environment and the stimuli they receive in the form of information; however, how to represent such adaptive processes is subject to heated debate. Several engineering and scientific disciplines, ranging from cultural anthropology to neurobiology, study how adaptive systems respond to stimuli. The approach taken by electrical engineers is based on a *dynamical systems* representation and branches into non-linear filtering, coding theory, and adaptive control. A relatively recent development, of interest to economics, has been the onset of neural networks and fuzzy systems as broad classes of “machine-intelligent” adaptive systems.

The approach adopted in this paper uses concepts from fuzzy control theory. An agent, instead of optimizing, follows Fuzzy Associative Memory (FAM) rules which map neighborhoods of “fuzzy” inputs to neighborhoods of “fuzzy” outputs, where fuzziness is defined by the degree of membership to more than one magnitude neighborhood (*e.g.* high, medium, low). Given input and output data, a set of rules (expressed by appropriate operations on the membership functions associated with each neighborhood) can be used to approximate any non-linear dynamic process. In general a FAM system encodes and processes in parallel a FAM bank of *m* rules. Each input to the FAM system activates each stored rule to a different degree. The output is determined by a weighted average of each rule output based on relative activation of the different rules. The rules can be determined *a priori,* based on expert knowledge, or they can be identified by adopting unsupervised learning algorithms. The mathematics involved, which is well laid out in Kosko (1992), maintains the flexibility of linguistic description while giving a consistent theoretical structure and empirical solution techniques.

The framework we are adopting is similar, in some respects, to *Classifier Systems* (CS) (Holland and Miller, 1991). Agents’ FAM rules are fired in parallel and the outcome is based on the output value membership (strength) associated with the input values. The fundamental difference between the two methods is that classifier systems choose a *single* action based on probability weights while FAM rules all activate to different degrees, thereby defining a spectrum of actions through averaging the rule activation. The implication of this difference is that, in contrast to fuzzy systems, CS systems always need many interacting agents to obtain a meaningful result (because they are tied to trials based on the probability weights). The analog of the adaptive component of CS is the adaptive FAM, which uses a product-space clustering procedure borrowed from neural network theory (adaptive vector quantization) that can weigh rules and discover new ones as the system evolves. In principle this approach could also be applied in the simulation of many interacting agents, as is done with the CS approach, but whether this would be computationally efficient is an open question.

Neural networks and fuzzy systems both estimate input-output functions, and both are trainable dynamical systems. Sample data shapes their reaction to input variables; however, unlike classical statistical estimators, they are ***model-free estimators*** in that they estimate a function without a mathematical model of how outputs depend on inputs. Neural and fuzzy systems encode sampled information in a parallel-distributed numerical framework.[7] Artificial neural networks, consisting of numerous, simple processing units programmed for global computation, have been applied to economics (Beltratti, 1996; Kaastra and Boyd, 1996; Rydygier, 1997).

Neural and fuzzy systems differ in how they associatively "infer", or map, inputs to outputs. Structured knowledge is represented in neural networks by a system of nonlinear functions that does not allow any interpretation of what was encoded during training. Unlike a language-based expert system, we cannot know which inferential paths a network uses to reach a given output. It appears that the price to be paid for model-free estimation with neural networks is system inscrutability. In the domain of economic policy analysis, presenting results without the theoretical intuition that goes with those results is considered unacceptable – the infamous "black box" syndrome of many simulation models. In contrast, fuzzy systems encode data in a format that can be easily interpreted linguistically (even though the method is numerical) and allow analysts to follow the inferential process. Before moving to the applied part of this paper, we introduce some mathematics of fuzzy systems.

Fuzzy Sets

Fuzzy system theory holds that all things are a matter of degree. A fuzzy set incorporates ambiguity.[8] At the heart of a fuzzy set is the violation of traditional laws of set theory such as non-contradiction {*not-(A and not-A)*} and the law of excluded middle {*either A or not-A*}. Mathematically, fuzziness means multivalence and was first developed formally in the 1920s by polish logician Jan Lukasiewicz (1970) by introducing a three-valued logical system and further extending the range of truth values {0,1/2,1} to all numbers in [0,1].[9]

[7] This sets these two methods apart from purely linguistic descriptions commonly used in artificial intelligence applications. The difference is important because, in symbolic processing, logical inference replaces time evolution and non-linear dynamics.

[8] A distinction has to be made between *randomness*, that describes the uncertainty surrounding the occurrence of a well-defined, and *event ambiguity* describing the degree to which an event occurs. Randomness and fuzziness differ conceptually and theoretically; however, they also share many similarities. Both systems describe uncertainty numerically in the unit interval [0,1] and they combine sets and propositions associatively, commutatively, and distributively. See Kosko (1992), chapter 7, "Fuzziness *vs*. Probability".

[9] Bertrand Russell (1923) first identified vagueness at the level of symbolic logic by analyzing classical paradoxes. He also arrived at a paradox of classical set theory that the set of all sets cannot itself be a set. A good description of this history is provided by Kosko (1997).

Although multi-valued logic had never, until recently, been used in economics, researchers well known in the economic arena such as Von Neumann (Birkhoff and Von Neumann, 1936) and Richard Bellman (Bellman *et al.*, 1966) contributed in the development of fuzzy set theory applied, respectively, to quantum mechanics and pattern classification.

In classical sets (or crisp sets), as represented by Venn diagrams, the process by which individuals from the universal set X are determined to be either members or non-members of a set can be defined by a *discrimination function*. For a given set A, this function assigns a membership value $m_A(x)$ to every $x \in X$ such that

$$m_A(x) = \begin{Bmatrix} 1 \ \ if \ \ and \ \ only \ \ if \ \ x \in A \\ 0 \ \ if \ \ and \ \ only \ \ if \ \ x \notin A \end{Bmatrix} \tag{1}$$

Thus, the function maps elements of the universal set to the set containing 0 and 1:

$$m_A(x): \ X \rightarrow \{0,1\}$$

Defining the operations of union ($\cup$), intersection ($\cap$), and complement ($\bar{A}$) the following well-known properties can be obtained: $A \cup \bar{A} = X$ (law of excluded middle) and $A \cap \bar{A} = \varnothing$ (law of non-contradiction).

On the other hand, a fuzzy set contains elements that have varying degrees of membership in the set and that can also be members of other fuzzy sets in the same universe. Elements of a fuzzy set are mapped to a universe of *membership values* using a function-theoretic form. This membership function, m_A maps elements of a fuzzy set **A** to a real numbered value on the interval [0,1], or equivalently:

$$m_A(x): \ X \rightarrow [0,1].$$

A common membership function, often used in applications, is the triangular set shown in Figure 1.

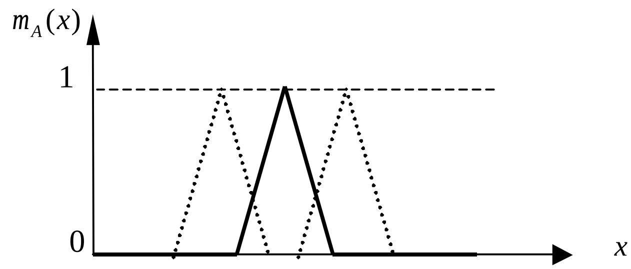


Figure 1. A possible membership function representation of a fuzzy set

Zadeh (1965) defines fuzzy set operations, which we can compare with operations on crisp sets:

Union $\quad m_{A\cup B}(x) = \max[m_A(x),\ m_B(x)]$ (2)

Intersection $\quad m_{A\cap B}(x) = \min[m_A(x),\ m_B(x)]$ (3)

Complement $\quad m_{\bar{A}}(x) = 1 - m_A(x)$ (4)

With these definitions, all properties of crisp sets (associativity, distributivity, transitivity, identity, idempotency, involution, and De Morgan's laws) apply to fuzzy sets except the law of excluded middle and the law of non-contradiction (Klir and Folger, 1988).[10]

From sets to inference

The next step is to define a procedure that will support inferential processes using fuzzy sets. To do this we must introduce the notion of *relation*, a basic idea behind numerous operations on sets such as Cartesian products, that may represent the presence or absence of association, interaction, or interconnectedness between the elements of two or more fuzzy sets. Relations represent mappings for sets and are also very useful in representing connectives in logic. It is exactly these two characteristics of relations that will prove the usefulness of the fuzzy system approach. On the one hand, mappings between sets can be treated as numerical constructs adhering to set-theoretic laws (mathematical functions being a sub-case of this approach), while on the other hand, a relation, R(X,Y), may be viewed as

[10] Note that when the range of membership grades is restricted to the set {0,1} these functions perform exactly as the corresponding operators for crisp sets, thus establishing them as a clear generalization of the latter.

an implication operation where an antecedent infers a consequent R(X,Y)= X→Y , allowing a logical interpretation of the mapping (at the set level this would be represented as $X \subset Y$).

Assume ***X*** to be the input set and ***Y*** to be the output set, a crisp binary relation ***R(X,Y)*** among these two sets can be expressed as a subset of the Cartesian product space ***X*** x ***Y***. Each crisp relation, just as other sets, can be described by a characteristic function:

$$m_R(x) = \begin{Bmatrix} 1 \;\; if \;\; and \;\; only \;\; if \;\; (x,y) \in R \\ 0 \;\; if \;\; and \;\; only \;\; if \;\; (x,y) \notin R \end{Bmatrix} \tag{5}$$

Similarly to how we defined fuzzy sets, we can define a *fuzzy relation* as a fuzzy set defined on the Cartesian product of crisp sets ***X*** x ***Y***, where tuples (*x,y*) may have varying degrees of membership within the relation. The membership grade is usually represented by a real number in the [0,1] interval and indicates the strength of the relation present between the elements of the tuple.

One possible implication operation, called *correlation-product encoding*, assumes that the degree of membership within an implication relation for fuzzy sets is given by the product of the separate memberships: $m_R(x,y) \; = \; m_A(x) \; \cdot \; m_B(y)$. This inferential procedure, commonly adopted in the fuzzy systems approach to engineering control problems, has the advantage over other methods of preserving more information (Kosko, 1992, p. 312). This is a crucial component of the Fuzzy Associative Memory (FAM) scheme. The FAM is a rule-based system that can provide an accurate numerical approximation of a dynamic system.

Fuzzy sets form the building blocks for fuzzy if-then rules such as “If unemployment is high then set a low interest rate”. The rules have the form “If X is A then Y is B” where A and B are fuzzy sets. A fuzzy system is a set of fuzzy rules that converts crisp inputs into crisp outputs. This is done in five steps:

(i) *Universe of discourse specification*. The input and output variables X and Y must be picked and fuzzy subsets of these variables defined.

(ii) *Fuzzy rule identification*. Relate the output sets to the input sets by identifying fuzzy rules.

(iii) *Fuzzification*. For a given input *x*, calculate each rule’s (R_i) antecedent strength as a fuzzy membership function $w_i \; = \; m_{A_i}(x) \cdot$

(iv) *Rule output evaluation*. Modify the output sets of each rule according to the correlation-product inference rule $m_{R_i}(y) = w_i \cdot m_{B_i}(y)$.

(v) *Defuzzification*. Conceptually, the approach is to form an output fuzzy set via the fuzzy union of individual rule outputs and then find its centroid (as in Figure 2). For particular cases, it is equivalent and computationally easier to defuzzify each rule output first and then combine the resulting crisp outputs So we perform centroid defuzzification for each output fuzzy set:

$$y_{R_i} = \frac{\int y\, m_{R_i}\, dy}{\int m_{R_i}\, dy} = \frac{\int y w_i\, m_{B_i}\, dy}{\int w_i\, m_{B_i}\, dy} = \frac{\int y\, m_{B_i}\, dy}{\int m_{B_i}\, dy} \tag{6}$$

The system output is then calculated by a membership-weighted sum of each rule's centroid value in the output space:

$$y = \frac{\sum w_i\, y_{R_i}}{\sum w_i} \tag{7}$$

There are many other prescriptions for fuzzification, rule evaluation, and defuzzification; however, the technique presented here is the widely used and easy to analyze.[11]

[11] The computational procedure for defuzzification explained above relies on the fact that we use correlation-product inference which also avoids the computation of integrals (assuming the fuzzy set centroids are known). Kosko (1997, p. 50) is a good reference for formal treatment of the Standard Additive Model (SAM) . Klir and Folger (1988) give a comprehensive overview of alternative prescriptions in fuzzy set theory.

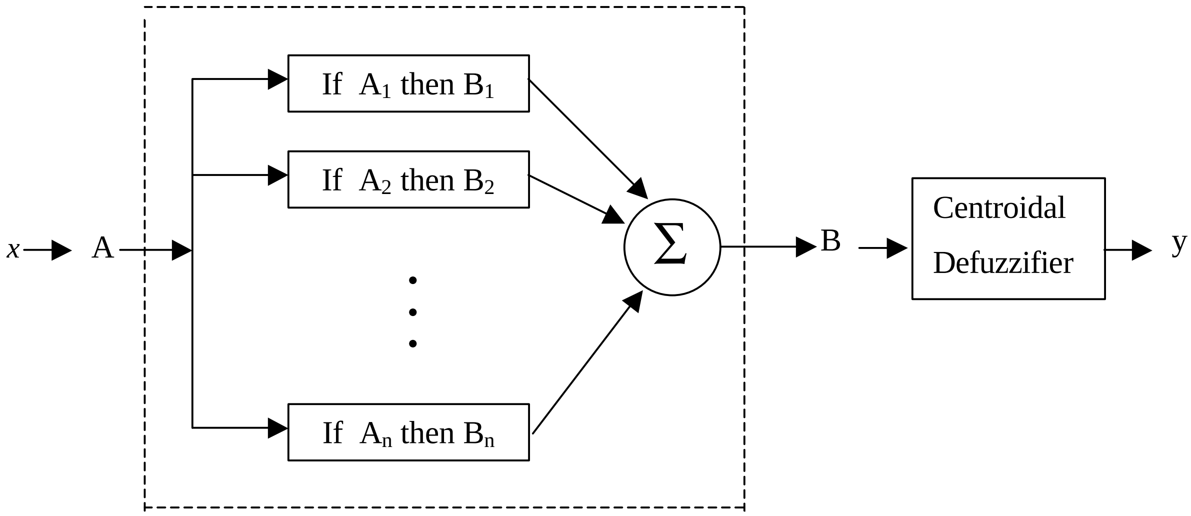


Figure 2. Fuzzy system architecture.

From inference to approximating dynamical systems

The same system presented in the previous subsection can be viewed as approximating the function that represents the true input-output relationship for the system in question. Additive fuzzy systems $F: \Re^n \rightarrow \Re^p$ can uniformly approximate any continuous function $f: U \subset \Re^n \rightarrow \Re^p$ assuming the domain U is closed and bounded (compact) (Kosko, 1997 p.127). This important result, known as the Fuzzy Approximation Theorem (FAT), can be understood intuitively if one considers that rules define fuzzy patches in the input-output space which are meant to cover the functions graph. The approximation tends to improve as the fuzzy rule patches grow in number and shrink in size as in Figure 3.

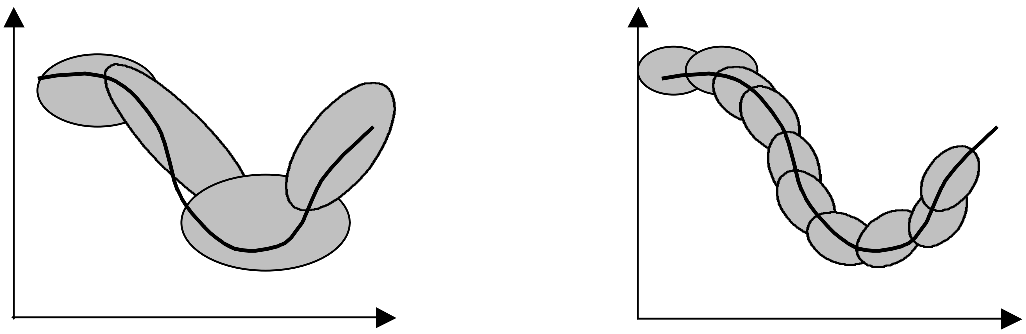


Figure 3. Fuzzy function approximation: patches in input-output space

Additive fuzzy systems average the patches that overlap by adding them. Then a centroid (or other operation) converts the patch cover to the function $F: \Re^n \rightarrow \Re^p$. It has been proved (Kosko 1997, pp 80) that all centroidal fuzzy systems F compute a conditional mean: $F(x) = E[Y \mid X = x]$. An additive fuzzy system splits this global conditional mean into a convex sum of local (rule) conditional means. The fuzzy system F does not use a math model of the system or function *f* that it tries to approximate. In this respect, *F* is a model-free statistical estimator; however, fuzzy systems implicitly implement a piecewise-linear interpolation between output values y_i (dependent on the amount of overlap between input fuzzy sets).[12]

The Economic Model

Consider a firm, with a given technology, facing fluctuating prices for its output. Following Dixit (1997), we assume that adjustment costs underlie the dynamics of the firm's factor demands. This problem has been traditionally addressed using optimal control

[12] In its most generic formulation, a "learning machine" is capable of implementing a set of functions $f(x,y)$, chosen a priori, before the formal inference process is begun (with parameters y indexing the set of functions). In the case of a standard parametric regression, the set of functions implemented may be $f(x,y) = \sum y_i x^i$. This approach can be used to characterize other techniques, such as non-parametric regressions, Fourier series, classification problems, clustering, and density estimation (Cherkassky and Mulier, 1998). All these techniques can be characterized by a taxonomy known as "dictionary methods" (Friedman, 1994), where a method is specified by given set of basis functions. For a characterization of fuzzy systems as a linear combination of normalized basis functions see (Cherkassky and Mulier, 1998).

techniques (either deterministic or stochastic) to solve for dynamic investment and employment decisions assuming adjustment costs are incurred by firms that change their capital stock and/or labor base. Dixit (1997) shows that, with linear adjustment costs, the dynamics exhibit a region of inaction. No adjustment is made unless its marginal value is sufficiently high. The introduction of uncertainty broadens this region of inaction. Dixit also identifies the characteristics of the boundary separating action from inaction, and considers what action should be taken in different sub-regions of the input space (Figure 4). This classification lends itself to an alternative representation as a FAM system, where the firm follows a set of rules that may or may not identify an optimal solution according to the Bellman formulation of the optimal control problem. The rules will reflect the technology available to the firm, and indicate regions of action and inaction linked to a price-responsive supply rule. The framework can be extended to include rules that consider systemic factors, such as interest rates or recession signals.

Although Dixit was the source of inspiration for the application in this paper, we do not address the issue of contemporaneous adjustment of both capital and labor. For this paper we address the simpler problem of single factor adjustment. We investigate, as do Nickell (1978) and Bentolila and Bertola (1990), employment dynamics with hiring and firing costs while holding capital constant (implicitly assuming that the costs of adjusting the capital stock are very high).

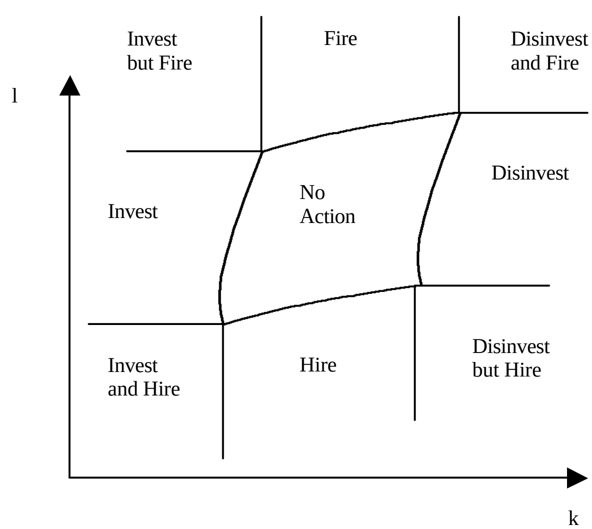


Figure 4 Inaction region and action boundaries (Dixit, 1997).

The variables *l* and *k* are the log-linear transformation of labor and capitol demanded by the firm assuming a Cobb-Douglas production function.

For clarity and tractability, it is easier to address single factor adjustments. In principle, however, the framework presented here could incorporate Dixit's full model. A further deviation from Dixit's approach is that we assume quadratic adjustment costs so as to force regularity on the sample generated by the optimal control model.

The variables considered in this application are:

- DPX: the change in price relative to the initial price condition (for normalization), and is specified as a moving average from t-1 to t+1 and centered around t;
- ADJC: adjustment cost of hiring or firing labor (unit cost of hiring and firing are assumed equal for presentational purposes)
- FDGAP: expresses the factor demand gap between current employment and static equilibrium employment as a share of current employment given current prices; a negative FDGAP entails the firm has an excess of employees if current prices were to persist.
- DLAB: hires or fires as percentage of current labor force; this is the decision variable

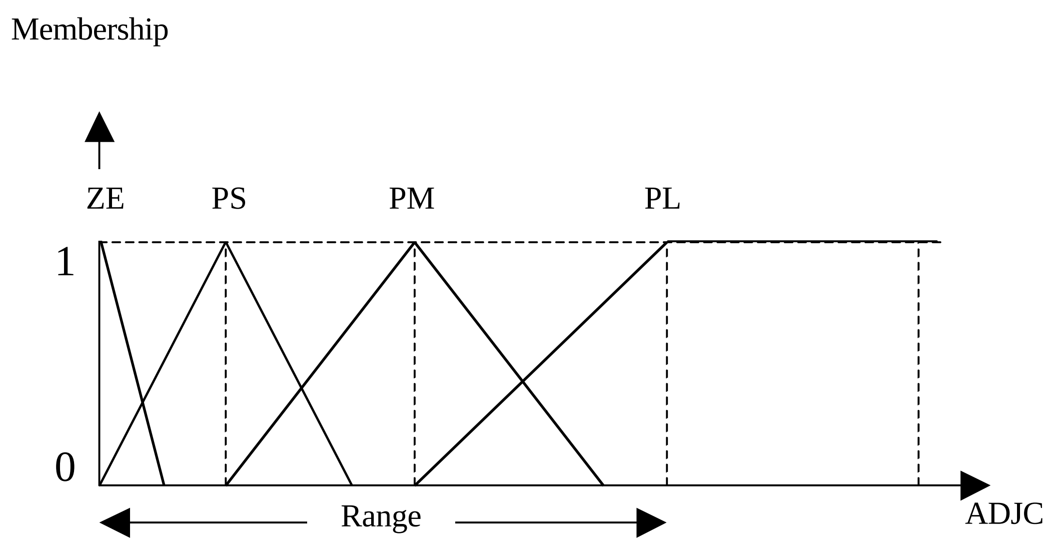


Figure 5. Membership function for labor adjustment cost

We partition the variable space using 4 overlapping sets for the non-negative variable ADJC, and 7 overlapping sets for the other three variables, DPX, FDGAP, and DLAB, which can have both positive or negative values. The membership functions are triangular except for those partitions representing the extrema. Figure 5 shows the membership functions for the partitions zero (ZE), positive small (PS), positive medium (PM), and positive large (PL).

Table 1 presents, for each fuzzy set, the point where the membership function peaks and the range (centered around the peak) over which the membership is non-zero. For example, the membership of the fuzzy set representing negative medium (NM) change in demand for labor (DLAB) peaks when DLAB takes on a value of –0.062 and has non-zero membership in a range of ±0.037 around the peak. The procedure to define the sets' support was simple: we took the range for each variable (as in Figure 5) as being defined by the minimum and maximum values in the data set and then divided this range and assigned each support requiring that the more far away from the origin, the wider the support (ZE is very tight compared to PL).

	Price Change DPX		Adjustment cost ADJC		Factor demand gap FDGAP		Labor change DLAB	
Neg. Large (NL)	-0.256	+0.110			-0.381	+0.169	-0.111	+0.049
Neg. Medium (NM)	-0.142	±0.085			-0.212	±0.127	-0.062	±.0.037
Neg. Small (NS)	-0.057	±0.057			-0.085	±0.085	-0.025	±0.025
Zero (ZE)	0	±0.028	0	±0.108	0	±.0.072	0	±0.022
Pos. Small (PS)	0.057	±..0570	0.215	±0.215	0.143	±0.143	0.045	±0.045
Pos. Medium (PM)	0.142	±0.085	0.538	±0.323	0.358	±..215	0.112	±.0.067
Pos. Large (PL)	0.256	-0.114	0.969	-0.431	0.644	-0.286	0.201	-0.089

Note on units: DPX is a proportional change of price relative to the initial price condition; ADJC is the adjustment cost normalized relative to initial adjustment cost; DLAB and FDGAP are expressed as a proportional change relative to the current values of employment and static equilibrium employment.

Table 1. description of the membership functions for all input and output variables

We assume that, from a procedural perspective, decisions are made according to a set of rules. Agents distinguish between a <u>transient state</u> in which prices are fluctuating and a <u>stationary state</u> for which DPX is zero or very close to zero. If prices are fluctuating, the firm decides whether to hire or fire employees based on the price derivative and the adjustment costs associated with hiring/firing. If prices are not changing, the firm calculates its static equilibrium employment given current prices, observes its current employment, and decides what to do based on the gap between the two (FDGAP). We are therefore considering the following types of rules:

(i) For the transient states in which prices are not stationary: **If** DPX is Positive Small (PS) and ADJC is Zero (ZE) **then** DLAB is Positive Small (PS)

(ii) For the stationary states: **If** FDGAP is Neg. Large (NL) and ADJC is Pos. Small (PS) **then** DLAB is Neg Medium (NM)

This approach is consistent with Heiner (1989) where conditions are derived for "imperfect" agents in a dynamic setting to converge to an optimal target. The necessary and sufficient conditions were obtained by allowing that agents cannot follow a sufficiently erratic optimal target x_t^* without error, and they form an estimate of its true position $\hat{x}_t$ called the perceived target. There are an infinite number of ways in which the perceived target can be defined to approximate optimal decisions: rules of thumb, imitation, extrapolation, and so on. In this paper, we distinguished between transient and stationary states of nature to which the imperfect agent reacts. We will see that the stationary FAM obtained through estimation will satisfy the main requirement for a dynamic system to converge to optimality, the requirement being that the action taken by the agent (FAM) is a partial adjustment toward the perceived optimal location.

To estimate rules, we first must have a sample. To address all the issues mentioned above, we construct a very special sample that is as consistent as possible with the positivist economics approach. We are trying to see if simple rules can replicate "as if" optimizing behavior; therefore, we *start from a sample obtained by running the optimal control problem under different product price scenarios*, which provides an ideal testing ground. If it cannot be done with such a sample, it probably cannot be done with real world data. In the next section, we present the optimal control model used to generate the sample and the estimation process that was used.

Sample Generation and the Estimation of Behavioral Rules

Our procedure is to generate a sample through the simulation of an optimizing, perfect-foresight firm. This sample is used to “train” a FAM system with no *a priori* knowledge of the rules. The objective of the exercise is to examine if the rule-based system can effectively mimic optimal, perfect-foresight behavior, and if so, under what circumstances. Conceptually, we can interpret the FAM training process as an agent applying Heiner’s reliability condition in deciding which rules to adopt. The general specification of the optimal control problem we used to generate the sample is the following:

$$MAX \sum_{t=1} \frac{\left[PX_t \cdot X_t - \sum_f WF_{f,t} \cdot FD_{f,t} - \sum_f ADJC_{f,t} \right]}{(1+r)^{t-1}} \tag{8}$$

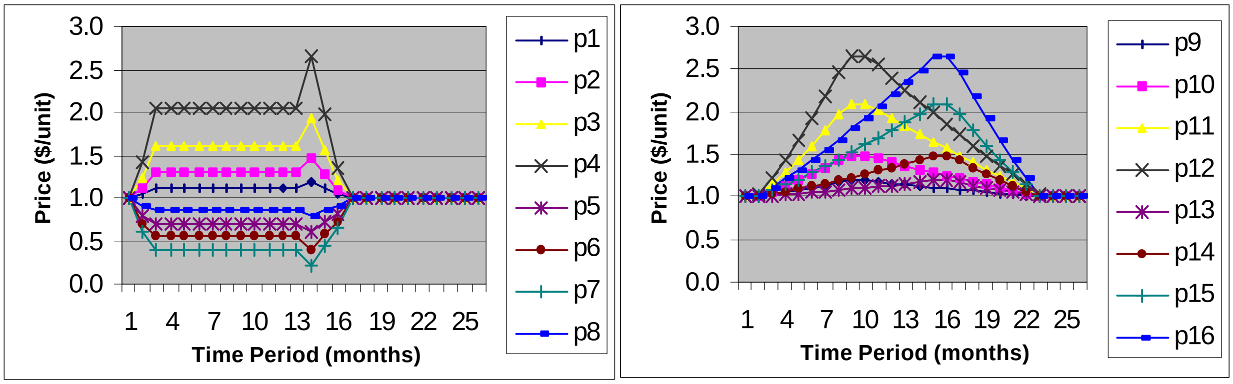
$$Subject\ \ to \quad X_t = \sum_f aFD^{a_f}{}_{f,t} \qquad \text{Cobb-Douglas technology } \left(\sum_f a_f = 1 \right) \tag{9}$$

$$FD_{f,t+1} = FD_{f,t} + FADJ_{f,t} \qquad \text{Factor Stock Equation of Motion} \tag{10}$$

$$ADJC_{f,t} \geq fh_f \cdot \left(FADJ_{f,t} \right)^2 \qquad \text{Quadratic Adjustment Costs} \tag{11}$$

where	PX_t	= product price	X_t	= production
	$FD_{f,t}$	= factor demand	$WF_{f,t}$	= factor wage
	$FADJ_{f,t}$	= factor adjustment	$ADJC_{f,t}$	= factor adjustment cost

We obtained the sample by providing the following 16 price paths to the optimal control version of the problem. One set of price paths was constructed to capture the reaction of the optimal control model to shocks allowing for an adjustment period (Figure 6a) while a second set of price paths was meant to extract optimal behavior in a smooth environment.



(a) **(b)**

Figure 6. Price paths imposed on optimal control model to obtain training sample.

Since we assume that the firm responds to adjustment costs, we ran the optimal control model using these price paths for 8 different values of labor adjustment costs in a range from 0.04 to 1.0 dollars per labor "unit" (we are assuming capital stock is fixed).[13] Wages are fixed at a value of 1 for both capital and labor, therefore an adjustment cost of 1 is high. The sample was obtained with 128 runs of the optimal control model over a 24 period time horizon (the time period can be viewed as a month for our purposes). More detail on the data generation process is available in the data appendix.

We estimated the FAM rules by adopting an *unsupervised learning* technique in which no distinction is made between input and output components of the training data (in contrast to supervised learning that does make the distinction). The objective is to approximate the unknown distribution that gave rise to the training sample using a small number of prototype vectors $C = \{c_1, c_2, \ \dots \ , c_m\}$ with *m* much smaller than the training sample size. The distribution is approximated by a collection of points (prototypes). The method adopted, called vector quantization (VQ), aims at minimizing a well-defined approximation (quantization) error when the number of prototypes *m* is fixed a priori. Vector quantization arose from the need to encode vectors in order to transmit them over digital communication channels (Gray, 1984; Shannon, 1959). The sample generation and the rule estimation were

[13] For convenience, the units were chosen to obtain unitary product price and wages.

both implemented with the GAMS software (Brooke *et al.*, 1998).

Since we assume that the firm reacts differently when prices are fluctuating as compared to when they are stationary, we divided the sample in two according to this criterion. We then proceeded to estimate one FAM for the transient and one for the stationary case. The vector quantizer adopted to estimate each FAM is a mapping of 3-dimensional Euclidean space R^3 into a finite subset $\boldsymbol{C}$ of R^3. Thus

$$Q: \; R^3 \longrightarrow C \qquad \text{where} \quad C = \{c_1, c_2, \ldots, c_m\} \text{ with } c_j \in R^3 \quad \forall j$$

We associate the *m* point quantizer in R^3 with a partition such that the regions defining it are non-overlapping and their union is the universe of discourse (in our case a subset of R^3). In our case, c_j takes on all values for which our fuzzy classes have maximum membership.

We performed repeated sampling among the 3,072 sample points (128x24) obtained for training, to obtain 25,000 points and assigned each one to an element of $\boldsymbol{C}$ by determining the c_j for which the sampled point obtained the highest membership.[14]

The economic justification of this procedure is intuitively provided by Heiner's result that behavioral rules arise because of uncertainty in distinguishing preferred from less-preferred actions. Assume the estimation represents the process by which a totally ignorant firm manager who has recently entered the business tries to understand when labor should be hired or fired based on price changes of the product produced by the firm. The sample can be viewed as data from the firm's successful competitors (success being a reasonable assumption in our case given the data were generated using a perfect foresight, optimal control model). Then the quantization process, by associating input-output regions that are found to most probably contain the actions of successful competitors, identifies reliable rules.

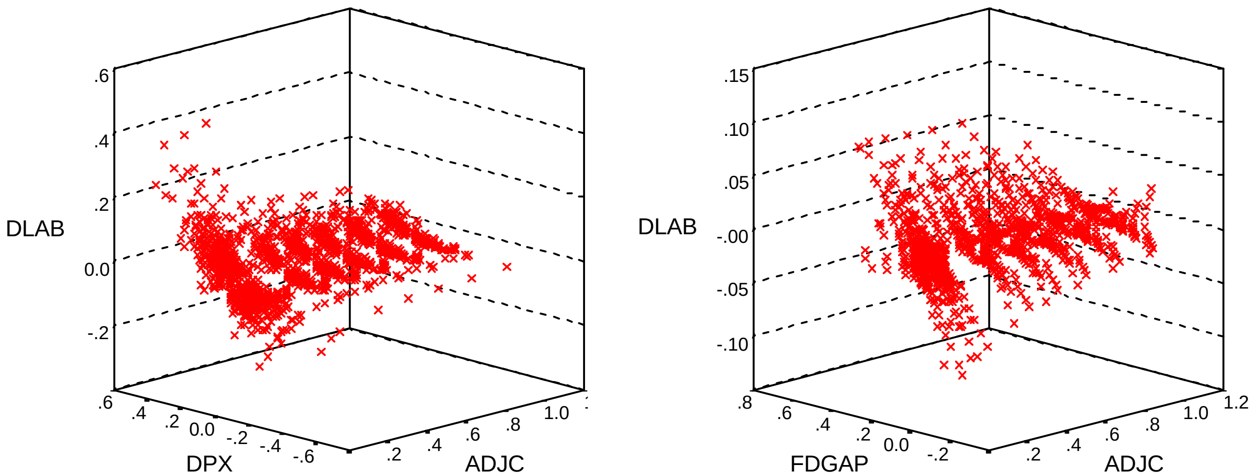
Given the particular nature of our sample, we can even use the reliability condition (Heiner, 1983). The question is to determine when the selection of an action is sufficiently reliable for an agent to benefit from allowing flexibility to select the action. We define *g(e)*

[14] This is a very elementary approach to a problem that has been studied extensively (see Gersho and Gray, 1992). The plain and basic approach used here ignores the necessary conditions for optimal vector quantizer design. The reason for such oversight is that we are interested in keeping this part as simple as possible for two reasons: (i) the simpler the rule estimation process, the more credible the fact that agents can identify such rules (different objective from designing an optimal signal processing system), and (ii) the focus of this paper is to demonstrate the feasibility of the rule-based approach as a whole rather than focus on technical issues that can be dealt with at a later stage.

as the gain from selecting the action when it is indeed preferred; *l(e)* as the loss from selecting it under the wrong conditions (where *e* represents the state of nature); right conditions for taking the action occur with probability $p(e)$, which are correctly recognized with probability r(U) (where U represents uncertainty faced by the agent); finally, the probability of taking the action at the wrong time is w(U). Following the above notation, Heiner's Reliability Condition is expressed as:

$$\frac{r(U)}{w(U)} > \frac{l(e)}{g(e)} \cdot \frac{1 - p(e)}{p(e)} \tag{12}$$

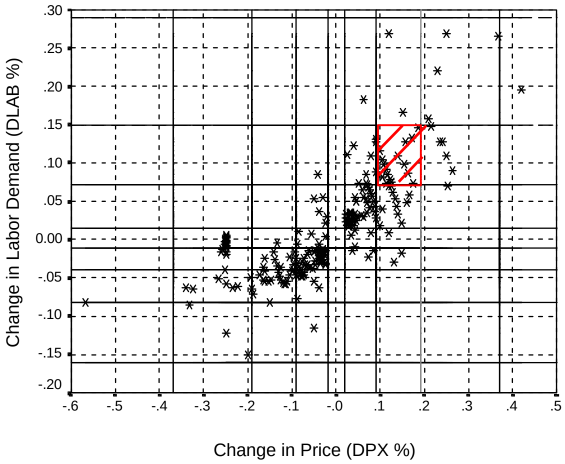
We next present, in terms of the variables to be adopted by the fuzzy system, the sample that was used to train the FAM rule based system. In Figure 7(a) the sample used to train the transient FAM rule was obtained by selecting all sample elements with fuzzy membership for non-zero values of DPX, while the remaining part of the sample (with DPX having membership in the fuzzy set ZE) was used to estimate the stationary FAM rule (Figure 7(b)). In Figure 6(a) the sample is presented in three dimensions with the price's rate of change (DPX) and the adjustment costs (ADJC) as input and the change in labor force as output, while in Figure 7(b) the inputs are adjustment costs (ADJC) and difference in labor demand (FDGAP) relative to a static optimum. One can see in both cases that the surface approximating the sample data is neither a plane nor other simple surface used in global estimation procedures. Without a reliable functional description (obtained either from theory or experience), methods such as a linear regression in parameters would not perform well.

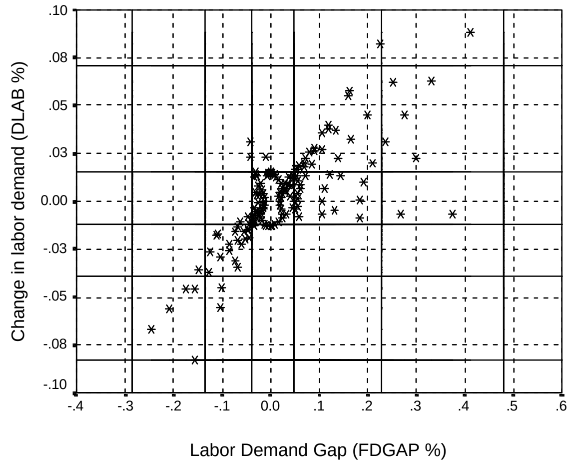


(a) **(b)**

Figure 7. Data sample generated using the optimal control model.

To understand the economic intuition of the rule estimation process, consider a two-dimensional world (for presentational purposes) and assume one input (DPX) and one output (DLAB) as in Figure 8(a). This allows us to link the reliability condition to the vector quantization process: the diagonally-shaded rectangle (assuming that the space was partitioned in rectangles as presented in Figure 8a) would be chosen by the VQ process as associating a change in price in the range [0.1,0.2] with a proportional change in labor force in the range [0.075,0.15].





(a) **(b)**

Figure 8. A two-dimensional cut of the sample spaces for the FAM rules

In this context, assume the input space to be partitioned over index *i* and the output space over index *j*, so that we can represent a rule partition by a set *R(i ,j)* associating input neighborhoods to output neighborhoods. *R(i ,j*) contains all possible rule associations; for any input partition P_i we denote *R*(i,j) as* the "winning rule", signifying the output partition Q_j has the highest number of sample elements for the given input partition. For convenience we define $P_i = \bigcup_j R(i,j)$ and $Q_j = \bigcup_i R(i,j)$ to represent, respectively, the i-th input and the j-th output partitions in input-output space. We also need to define $P^*(i,j) = \bigcup_{i \in R^*(i,j)} P_i$ describing, for each output partition *j*, the region in input-output space that will be categorized by the "winning rules" as belonging, in output, to the Q_j neighborhood. Then the components of the reliability condition can be expressed as:

$$p_j(e) = \frac{\text{number of sample elements in } \mathrm{Q_j}}{\text{total sample size}} \tag{13}$$

$$r_j(U) = \frac{\text{number of sample elements in } \bigcup_{\mathrm{i}} \mathrm{R^*(i,j)}}{\text{number of sample elements in } \mathrm{Q_j}} \tag{14}$$

$$w_{\mathrm{j}}(\mathrm{U}) = \frac{\text{number of sample elements in } \bigcup_{\mathrm{i}} \left[\mathrm{P^*(i,j)} \cap \overline{\mathrm{R^*(i,j)}} \right]}{\text{number of sample elements in} \bigcup_{\mathrm{i}} \mathrm{P^*(i,j)}} \tag{15}$$

It is important to remember that *the action*, in Heiner's sense, is given in our framework by the value of the output variable (this is why the LHS of equations 13-15 are indexed over *j*). Equation (13) expresses the probability that an action in the range of output partition Q_j is warranted. Once the "winning" rules have been chosen, the probability of our set of rules correctly recognizing that, in fact, action in the range of output partition Q_j is required is expressed in Equation (14). Finally, Equation (15) considers the probability of our rules incorrectly choosing an action in the Q_j range as depending on the sample points that fall in the input domain of the rules that choose Q_j but whose output value is not in the Q_j range (numerator) and on all the sample points that fall in the input domain of the rules that choose Q_j (denominator).

While *l(e)* and *g(e)* cannot be deduced from the diagram, supposedly the firm manager has a vague idea about the order of magnitude of the gain from a correct decision and the opportunity cost of an incorrect decision. Define the operator $\|R(i,j)\|$ as representing the number of sample elements in partition R(i,j). We can therefore manipulate the reliability condition to become

$$\frac{g(e)}{l(e)} > \frac{w(U)}{r(U)} \cdot \frac{1 - p(e)}{p(e)} \tag{16}$$

Assuming a fixed environment (e) and uncertainty faced by the agent (U), we index over the output partition (since each output variable range is associated with a different action), and we obtain:

$$\frac{g(j)}{l(j)} > \frac{\left\|\cup_i \left[\mathrm{P^*(i,j)} \cap \overline{\mathrm{R^*(i,j)}}\right]\right\|}{\left\|\cup_i \mathrm{P^*(i,j)}\right\|} \cdot \frac{\|\mathrm{Q_j}\|}{\left\|\cup_i \mathrm{R^*(i,j)}\right\|} \cdot \left\{\frac{\|\cup_i \mathrm{P_i}\|}{\|\mathrm{Q_j}\|} - 1\right\} \tag{17}$$

This can be viewed by the "learning" manager, if the sample comes from real world data, as an indication of trade-offs associated with actions.[15] More precisely, if successful competitors are adopting an action, then the ratio *g(e)/l(e)* must be favorable to this action,

[15] Inequality 17 expresses a global criterion for a rule set. An alternative, local expression that applies to single rules (rather than an action in output space that may be activated by several different rule antecedents) is given by: $\frac{g(i,j)}{l(i,j)} > \frac{\|\overline{\mathrm{R^*(i,j)}}\|}{\|\mathrm{P_i}\|} \cdot \left\{\frac{\|\mathrm{P_i}\|}{\|\mathrm{Q_j}\|} - 1\right\}$ which is obtained by calculating $p_{i,j}$, $r_{i,j}$, and $w_{i,j}$ when restricting the sample size to include only those elements whose input values are included in partition *i*. Inequality 17 is a generalization of the above expression.

which means it must be at least as good as the right-hand-side (which the “learning” can compute).[16] The intuition we can derive from this expression is that the more widely scattered the sample elements the higher must be the *g(e)/l(e)* ratio for an action to be undertaken.

The stationary and the transient FAMs obtained with this method are presented below.

<u>Stationary FAM</u>

- As adjustment costs decrease, the firm will tend to move faster toward a stationary equilibrium solution.
- If adjustment costs are very high, you may want to hire but you never fire.
- There is a relatively large region of inaction for positive values of FDGAP, implying that if adjustment costs are medium to high, the firm may never reach the stationary equilibrium position.

[16] Although the expression obtained is complicated to read, it is quite easy to compute with the division operation being the most computationally demanding.

Table 2. Stationary FAM

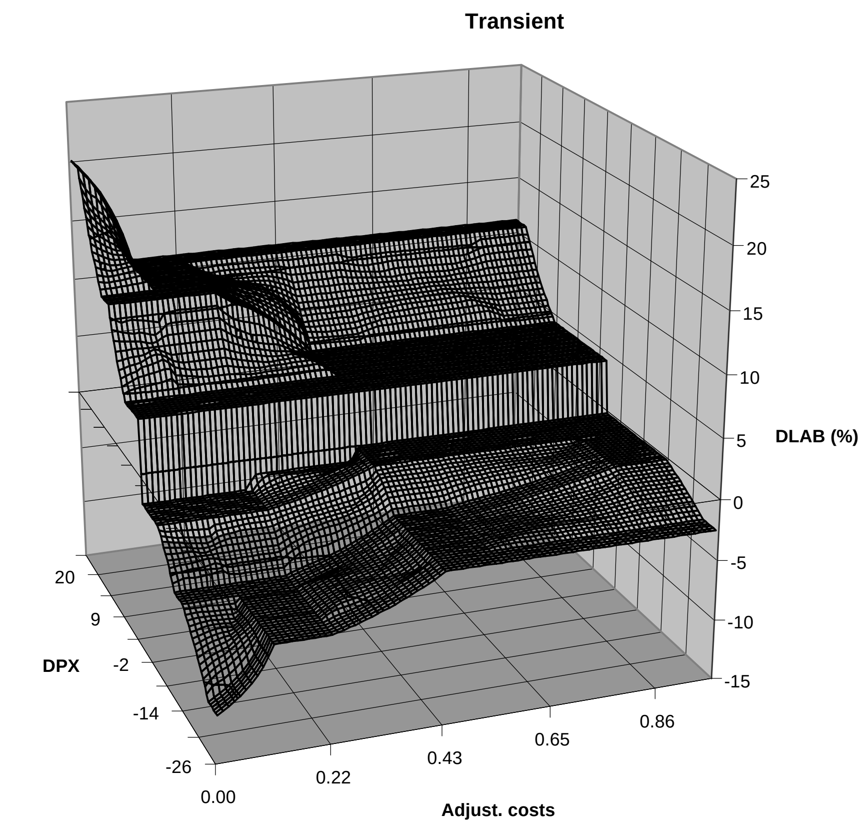
		Adjustment Costs			
		ZE	**PS**	**PM**	**PL**
Distance to stationary optimum labor demand	**NL**	NL	NM	NS	ZE
	NM	NL	NM	NS	ZE
	NS	NS	NS	ZE	ZE
	ZE	ZE	ZE	ZE	ZE
	PS	PS	PS	ZE	ZE
	PM	PM	PS	PS	PS
	PL			PS	PS

Transient FAM

- The area of inaction is relatively small.
- Higher adjustment costs dampen the reaction to price changes.
- There is a large area for which a small increase in labor force is the decision (+5%).

Table 3. Transient FAM

		Adjustment Costs			
		ZE	**PS**	**PM**	**PL**
Change in Price	**NL**	NL	NM	NS	NS
	NM	NM	NM	NS	ZE
	NS	NS	NS	ZE	ZE
	ZE	---------	---------	---------	---------
	PS	PS	PS	PS	PS
	PM	PM	PM	PS	PS
	PL	PL	PM	PM	PM



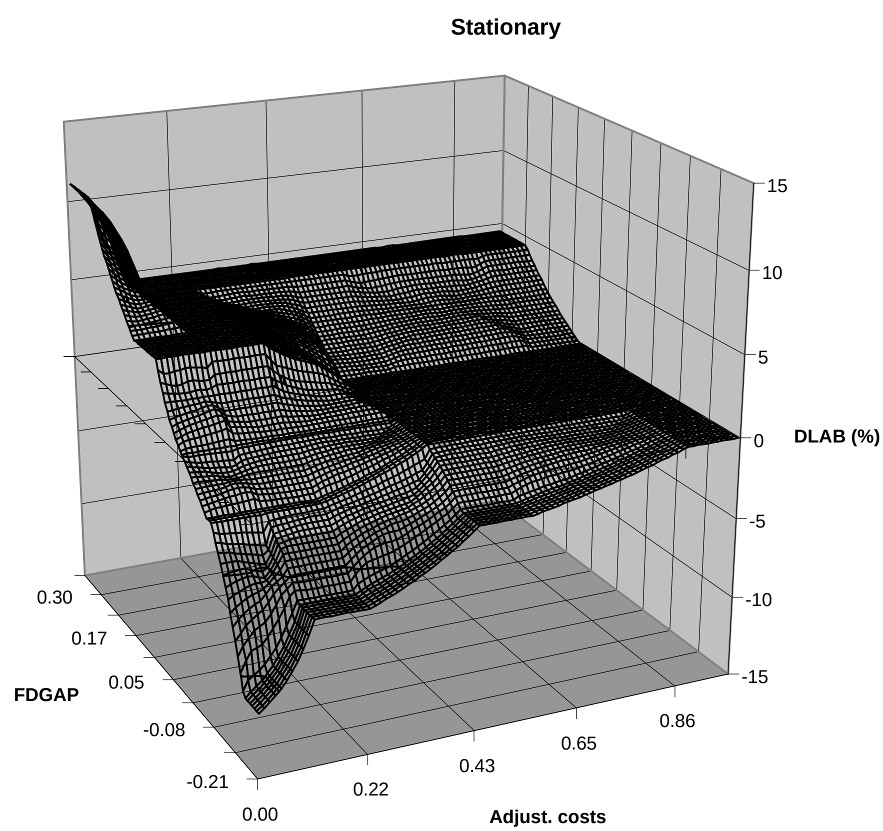


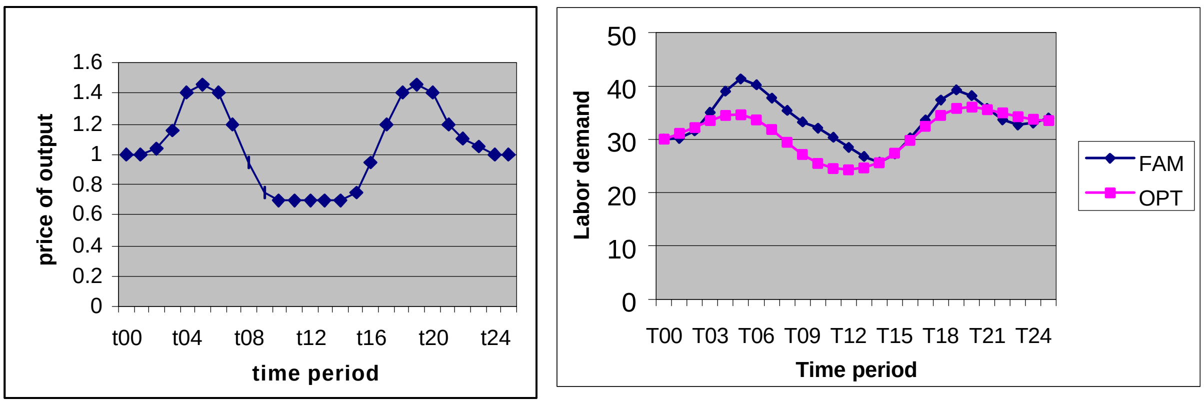
Figure 9.. Estimated FAM surfaces for Transient and Stationary FAM

The surfaces presented in Figure 9 represent the estimated FAMs with the inputs on the horizontal axes and the output labor decision on the vertical axis. These can be compared with Figure 7, which showed all the elements in the original sample (before doing repeated sampling) used in the estimation process. The end result of the estimation bears a good resemblance to the sample.

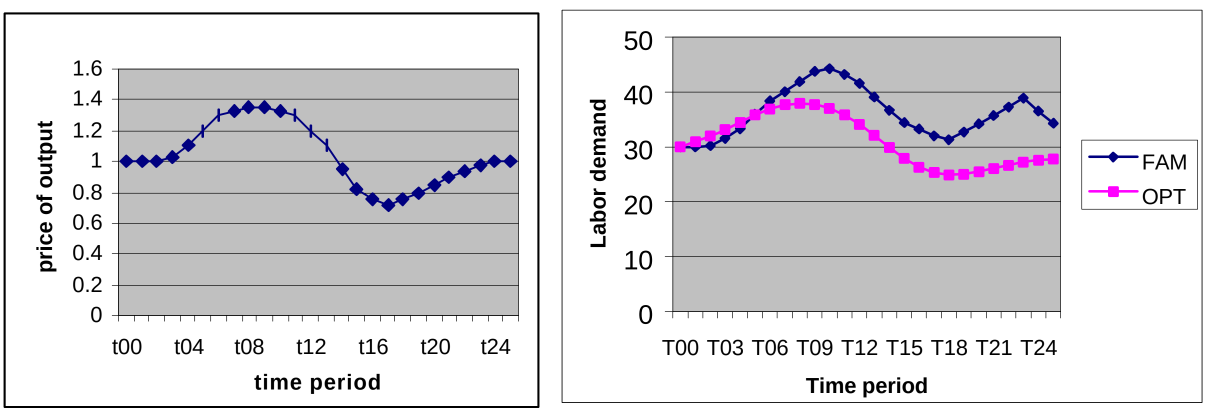
An important observation is that the change in labor demand in percentage terms is always less than the perceived target to reach a stationary solution (FDGAP), implying a partial adjustment process as required by Heiner (1989) to guarantee convergence to an equilibrium optimal solution. However, given the nature of our problem, at high adjustment costs, the FAM may never converge to the static optimum solution because of the presence of a region of inaction (Heiner assumes that partial adjustment is strictly non-zero, and at higher adjustment costs we violate this hypothesis). This is a different issue from the fact that it may, in fact, not be *dynamically* optimal to converge to the static equilibrium as pointed out in Dixit's region of inaction in his optimal control version of the problem.

FAM Performance in Predicting System Behavior

To test how the FAM performed tracking the system it was designed for, we chose four price paths: the first two paths simulate smooth business cycles, while the third and fourth represent, respectively, a permanent price increase and decrease. The test is performed assuming a low (0.15) adjustment cost.



(a) PATH 1

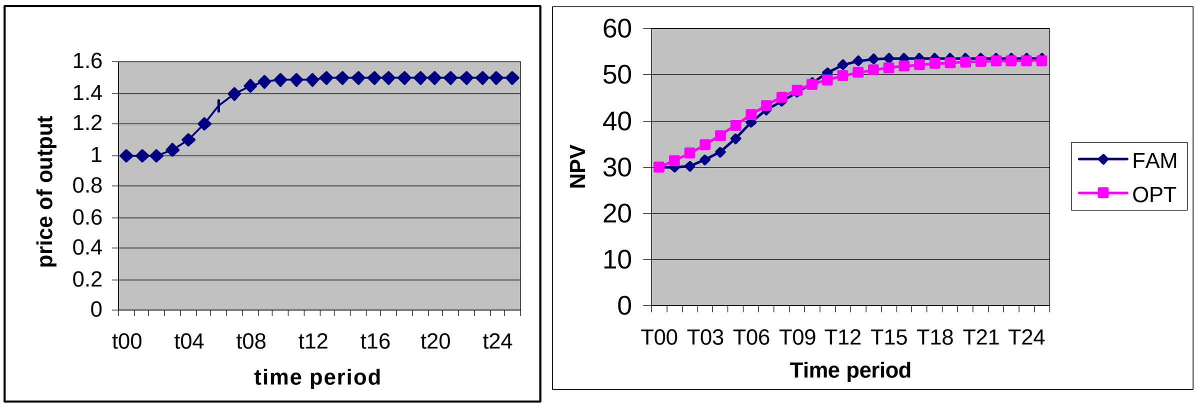


(a) PATH 2

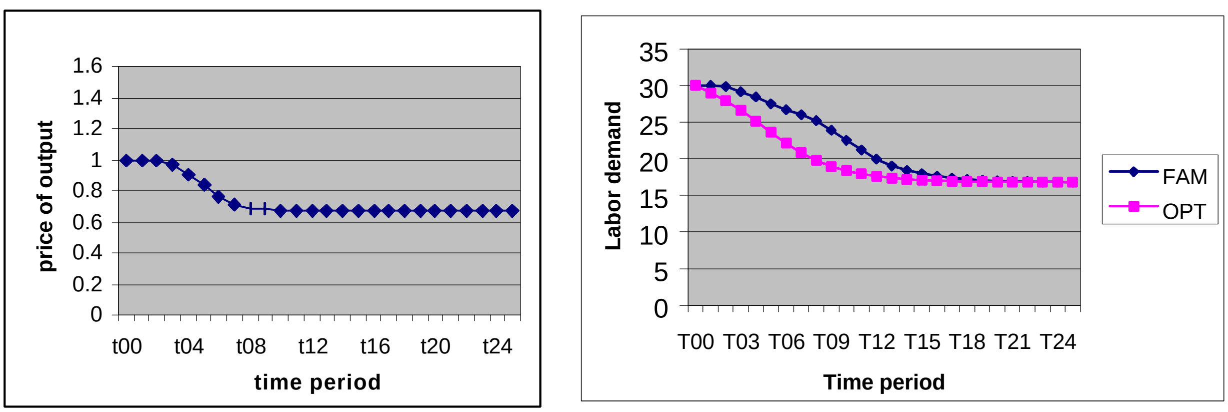
Figure 10. Business cycle price paths and comparison of FAM and optimal labor demand PATH 1 and PATH 2 represent cycles with different frequencies

The results show that the FAM performs well in both the business cycle situation and the permanent change scenarios (Figures 10 & 11). Figure 10 shows that the FAM at low adjustment costs tends to overshoot when there are price increases; however, the overall labor demand by the firm is well replicated.

The permanent price change scenario is tracked very precisely by the FAM, as can be seen in Figure 11. The FAM performs slightly better in the price increase scenario than in the price decrease one.



(a) PATH 3



(b) PATH 4

Figure 11. Permanent real price changes and comparison of FAM and optimal labor demand PATH 3 and PATH 4 represent permanent increase and decrease respectively

The first objective, to show that an appropriately trained rule-based system can behave as if it were an optimizing agent, has been accomplished at least in conditions of smooth price changes. Having established that there is consistency with the data for both the rule-based and the optimizing agent approaches (the latter by construction), in the next section we examine the two approaches under varying levels of noise affecting the price fluctuations.

Rules and Optimality Under Noisy Conditions

We now analyze the behavior of the FAM in the presence of differing degrees of noise, and compare it to the solution of a perfect foresight, optimal control solution. The comparison is made from the firm manager's perspective (who adopts the rules) by examining the difference in net present value of the flow of profits throughout the whole period represented by the simulation.

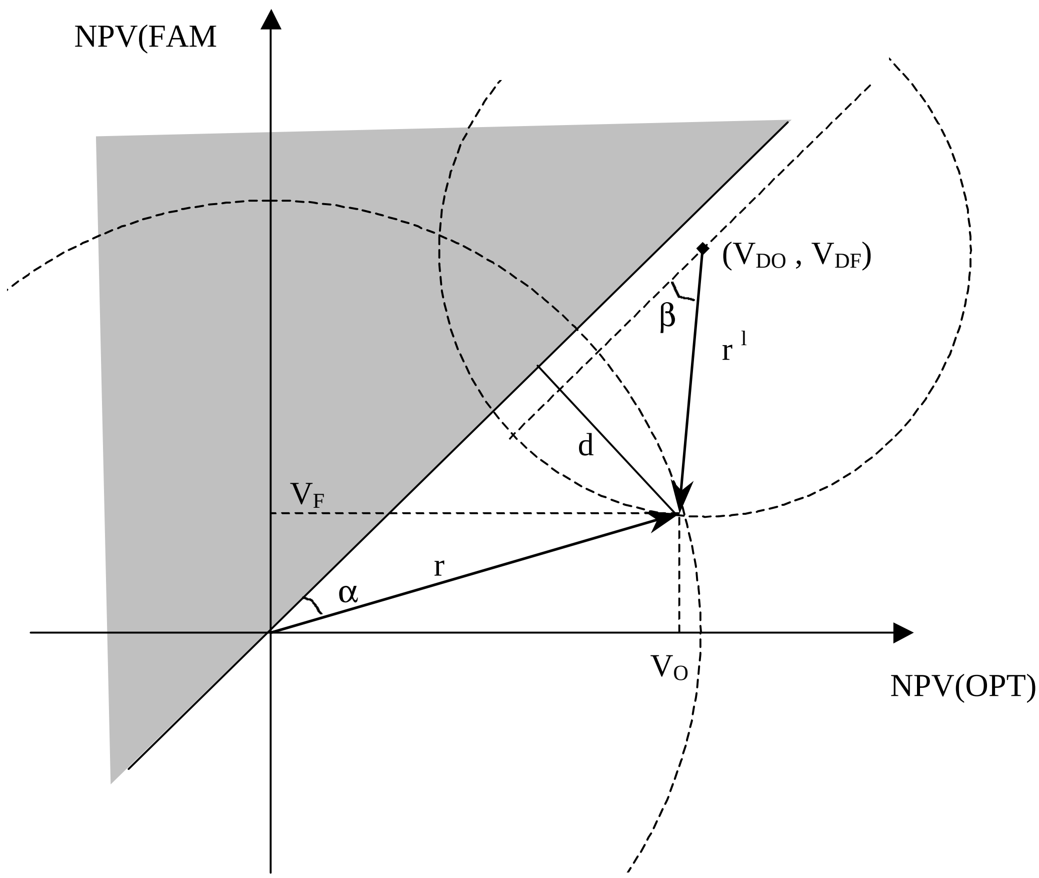


Figure 12. Geometric comparison of FAM vs. Optimal Performance

Figure 12 gives a geometric description of the measures we use to represent the performance of the FAM system relative to that of the optimal control system. The diagram plots the net present value of the flow of income from the optimal control solution on the x-axis (V_O) and from the FAM on the y-axis (V_F). For every stochastic price path, there will be a point on the xy-plane comparing these two values. The point (V_{DO} ,V_{DF}) represents the net present value for the optimal solution and the FAM solution in a deterministic, no-noise environment. The 45° line describes situations in which the FAM performs as well as the optimal control model. The half-plane above the 45° line is empty because the FAM can never outperform what is a *de facto* rational expectations model. In fact, the no-noise FAM solution is slightly below the 45° line, as would be expected.

An obvious measure that comes to mind for comparing performance of the two systems is the *Net Worth Gap* (NWG) expressed as the difference in Net Present Value between the optimal solution and the FAM solution. In Figure 12 the minimum distance from the 45° line to the point (V_O ,V_F), or *d*, represents the net worth gap (normalized by a factor of $\sqrt{2}$).[17] Problems arise with the NWG as a measure of performance because its value is unchanged for all solutions that fall on a parallel line with 45° slope. The NWG does therefore not take into consideration the baseline net worth of the firm, implicitly ignoring that losing a million dollars is very different when it happens to a family business rather than to a multi-national corporation.

To prevent the shortcomings of the NWG as a measure of performance, we can normalize it relative to the net worth of the firm. A first option is to define a norm, *NORGAP*, as the minimum distance (*d*) of the point from the 45° line over the distance from the origin (*r*).[18] It is therefore equivalent to taking the sine of the angle (α) between the radial through the point and the 45% line (*Sine(a)* is always positive in the interval (0,180)). The larger the value of the norm, the worse the performance of the FAM. Comparing the points along the half-circle with radius *r*, the FAM performs most poorly when passing the perpendicular to the 45% line. In this situation the value of the norm equals one (d=r) and the optimal control model has a positive NPV while the FAM has a negative NPV of the same magnitude.

An alternative measure of performance can also be introduced by normalizing NWG relative to the noise-induced deviation from the deterministic solution obtained in the absence of noise. Such a measure, that we call “Deviation Gap” (*DEVGAP*), can be obtained

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[17] From the geometric representation we obtain that $d = \frac{|V_O - V_F|}{\sqrt{2}}$

[18] Given the tuple (V_O , V_F) associated with a stochastic price scenario, the Euclidean distance *r* represents the Net Worth averaged geometrically over the two solutions (optimal and FAM).

as the minimum distance (d^l) of a point from the 45° line passing through the deterministic solution over the distance from the deterministic solution (r^l).[19] It is equivalent to taking the sine of the angle β. The sign of the DEVGAP measure is amenable to interpretation: a negative value of DEVGAP for a tuple (V_O ,V_F) signals that the FAM performance, in the presence of noise, comes closer to optimality than in the deterministic case; conversely, a positive sign implies that the FAM is further from optimal behavior. In the discussion that follows, both NORGAP and DEVGAP are used as measures of performance and the results according to these two measures are compared.

Figure 13 plots the net present value according to the approach just discussed. We observe that the performance of the FAM in the presence of noise depends on the different price paths.

[19] Given the tuple (V_O , V_F) associated with a stochastic price scenario, the Euclidean distance r^l represents the noise-induced deviation of Net Worth averaged geometrically over the two solutions (optimal and FAM).

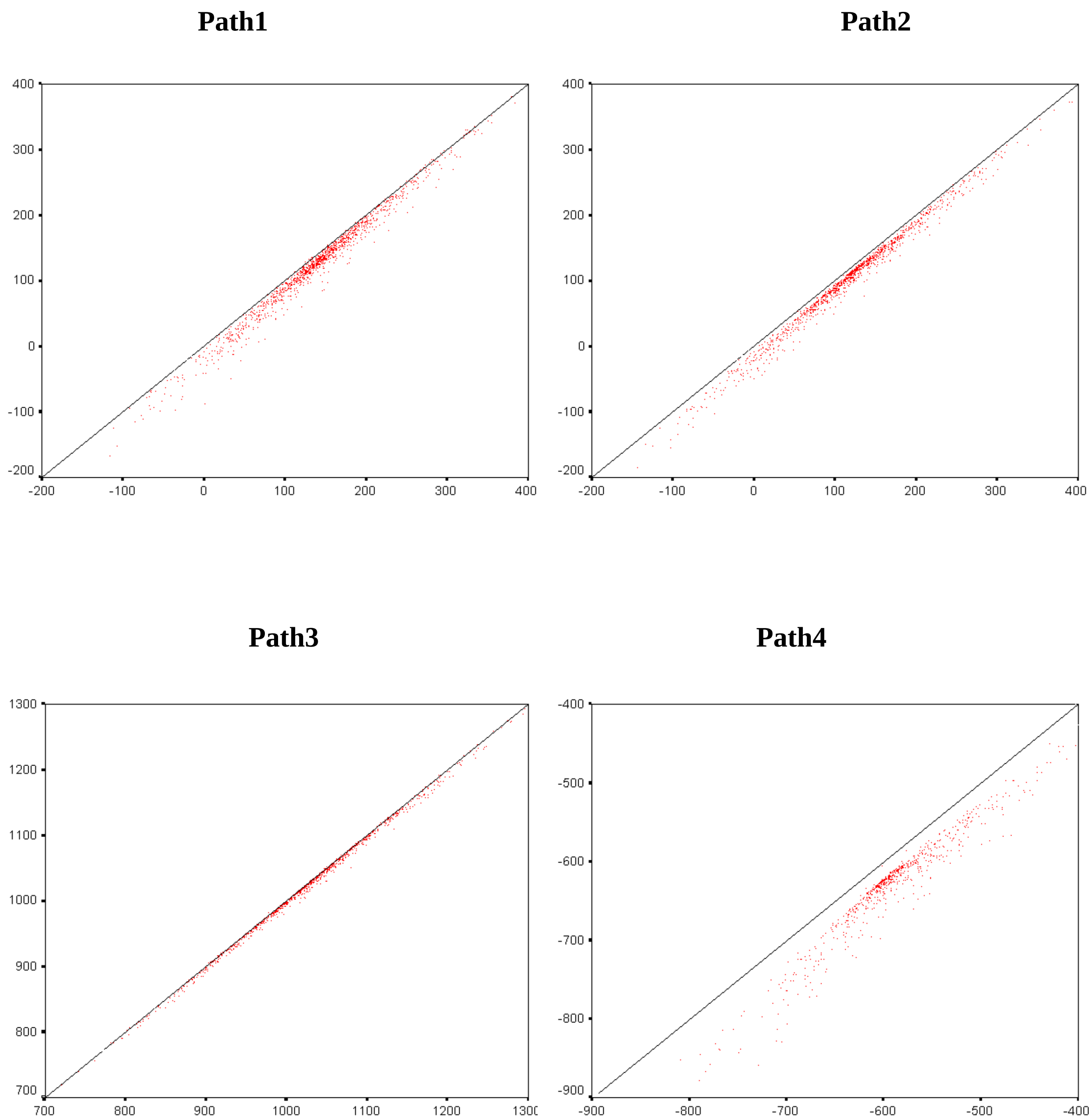

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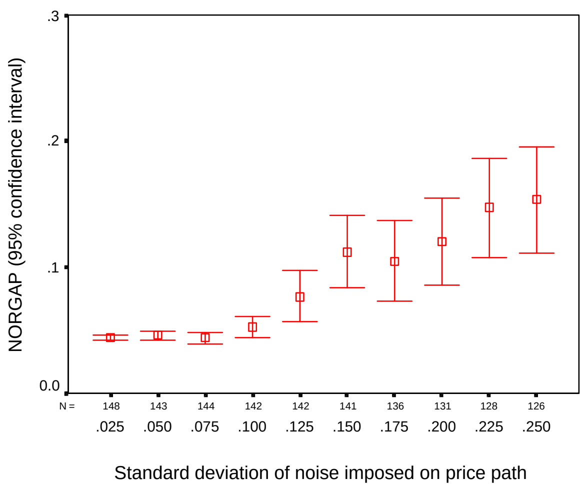
Figure 13. NPV ($) comparison of FAM vs. Optimal performance (x-axis: optimal, y-axis FAM)

When the price signal is corrupted by noise, the best approach for understanding the different performance of the FAM relative to the optimal control case for each price path is to construct a measure of the magnitude of the noise relative to the signal. The simplest approach, since we assumed additive white noise, is to use the variance of the noise and see how it affects the performance of the FAM. In Figure 14, we present the Net Worth Gap normalized relative the distance from the origin (NORGAP). On the x-axis is the standard deviation of the noise process affecting our price signal while on the y-axis we have the mean value of *d/r* and the 95% confidence interval for the mean.

The results show that the business cycle cases deviate more from optimality as a whole than do the permanent price change scenarios (Figures 14 and 15). This result is due to the fact that the deterministic starting point for the "permanent change" price scenarios are far away from the origin in the geometric representation (and close to the 45% degree line); therefore, the extent to which the FAM deviates from optimality is attenuated by the large value of *r*.

Path 1



Path 2

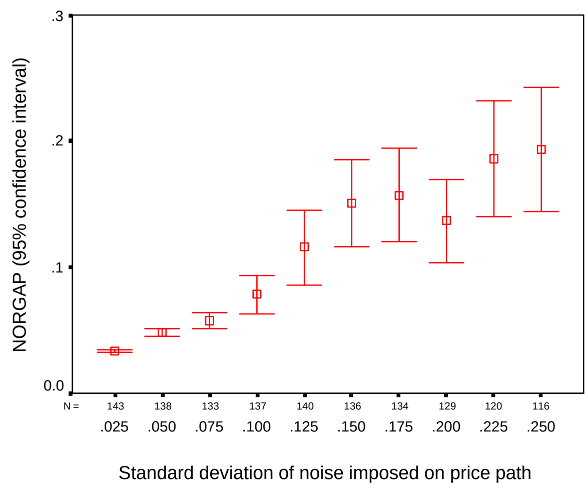


Figure 14. The 95% confidence interval for the normalized net worth gap ratio (NORGAP) at different standard deviations (Business cycle scenarios)

The results also show, as expected, a trend toward greater deviation from optimality as the variance increases. Although no general statement can be made, the threshold effect present in the response to Path1 is of interest in understanding where the *as if* optimizing hypothesis may break down in the presence of noise. More precisely, we notice that for a standard deviation up to 0.1 the FAM performs in a 5% range from optimality, but rapidly deteriorates to a 15-20% deviation from optimal behavior as the standard deviation increases.

The difference between the FAM response in the two permanent price change scenarios can be attributed again to the relative distance of the deterministic NPV from the origin being much higher in the case of the upward price adjustment case.

Path 3 **Path 4**

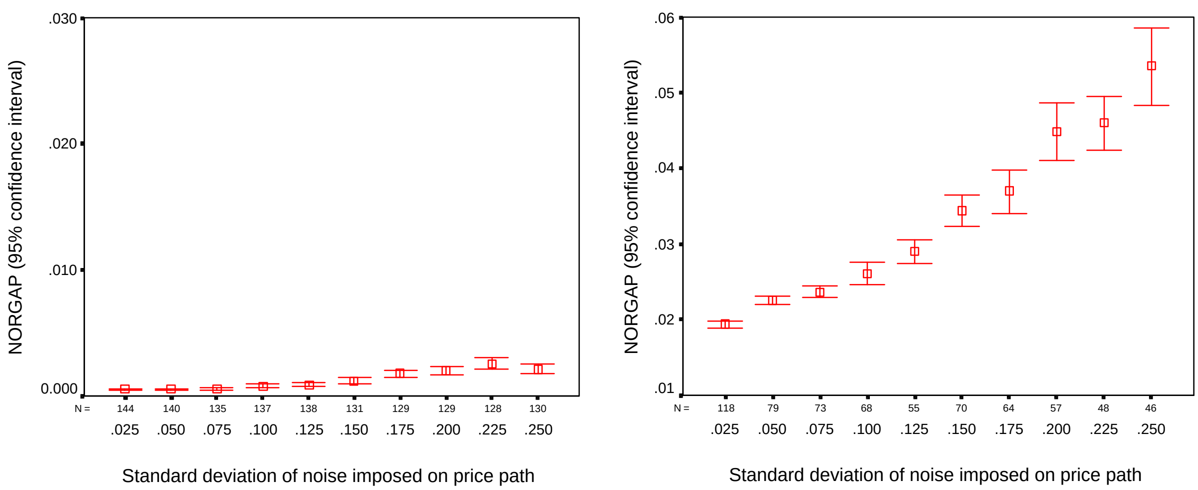


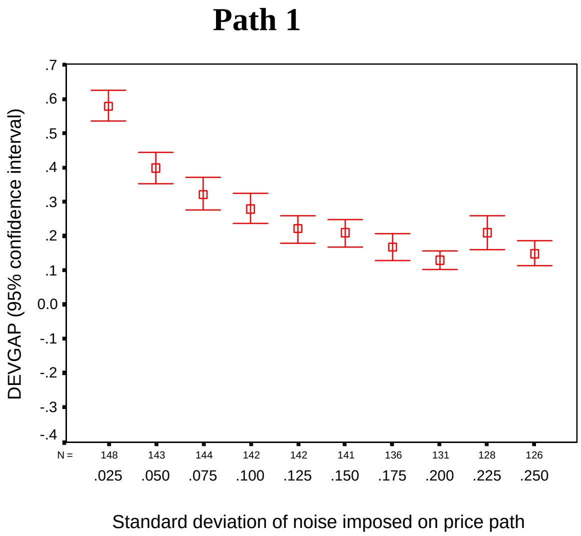
Figure 15. The 95% confidence interval for the normalized net worth gap ratio (NORGAP) at different standard deviations (permanent price change scenarios)

The results look very different if, instead, we adopt the deterministic case as a counterfactual. Below we present the measure obtained relative to the deterministic solution (DEVGAP). From Figures 16 and 17 we observe, as the noise level increases, a general tendency for the rule-based method to become a better approximation of optimal behavior if the reference point is taken to be the deterministic solution. At low variance levels, the FAM appears to be very sensitive to noise: a DEVGAP value of 0.6 implies an angle of 47°

relative to the reference line at 45°, which implies that either the optimal response is shifting and the FAM is not following, or the FAM is overreacting to the noise and the optimal response remains close to deterministic (reasoning is based on Figure 10, assuming the deterministic solution is now the origin: an angle of 45° to the 45° line implies that we are moving either along the x-axis or the y-axis). As it turns out (as we will see from analyzing the labor response), the FAM is overreacting at low noise levels. What is quite extraordinary is the relative improvement in performance as the noise level increases for the scenarios found to be sensitive at low noise. This result is in stark contrast with the preceding analysis centered around zero NPV as origin.

An exception to the relative improvement of the FAM rules under noisy conditions is given by the second of the business cycle scenarios (PATH2 in Figure 16). What distinguishes this scenario from the others is the relatively poor performance of the FAM solution in the deterministic case compared to the optimal. This means the point (V_{DO} ,V_{DF}) is further away from the 45° line bisecting the x-y plane than for the other scenarios. The negative value intervals for DEVGAP at low noise levels indicate an improvement in absolute terms of FAM performance expressed by (V_O ,V_F) being closer to the 45° line than the deterministic solution (V_{DO} ,V_{DF}) .

What is the economic intuition behind the results according to the different measures? It appears that in absolute monetary terms, if the deterministic FAM solution is close to optimal, the performance of the FAM deteriorates as noise increases. However, the deterioration, relative to normal deterministic conditions, is less than proportional to the increase in noise. Stated differently, the more unpredictable are prices, the more robust the rule-based system becomes by moving away from “deterministic normality” in a direction that, on average, tends towards optimality. In a very noisy environment, there is an upper limit to how much the FAM will stray from optimality. This is best seen in Figure 16 where at high noise levels the 95% confidence interval is in the range (0,0.2) meaning the angle β is between 0° and 11.5°.



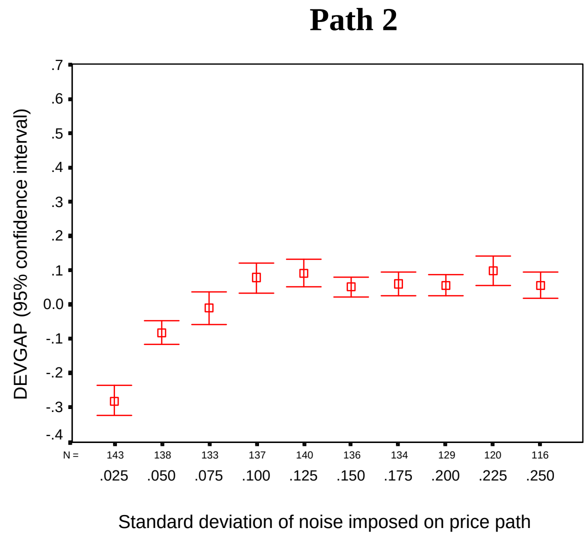


Figure 16. The 95% confidence interval for the net worth gap ratio normalized relative to the no-noise net worth (DEVGAP) at different standard deviations (Business cycle)

In Figure 17 we observe, as the noise level increases, that the FAM solution becomes a better approximation of optimal behavior if the reference point is taken to be the deterministic solution; however, the upper limit in terms of FAM deviation is different in the two cases shown. In the first case (PATH3) the noise is superimposed over a permanent price increase while in the second case (PATH4) the same noise is superimposed over a permanent price decrease. The difference between the two price scenarios leads to different signal-to-noise ratios which affect the performance of the FAM system.

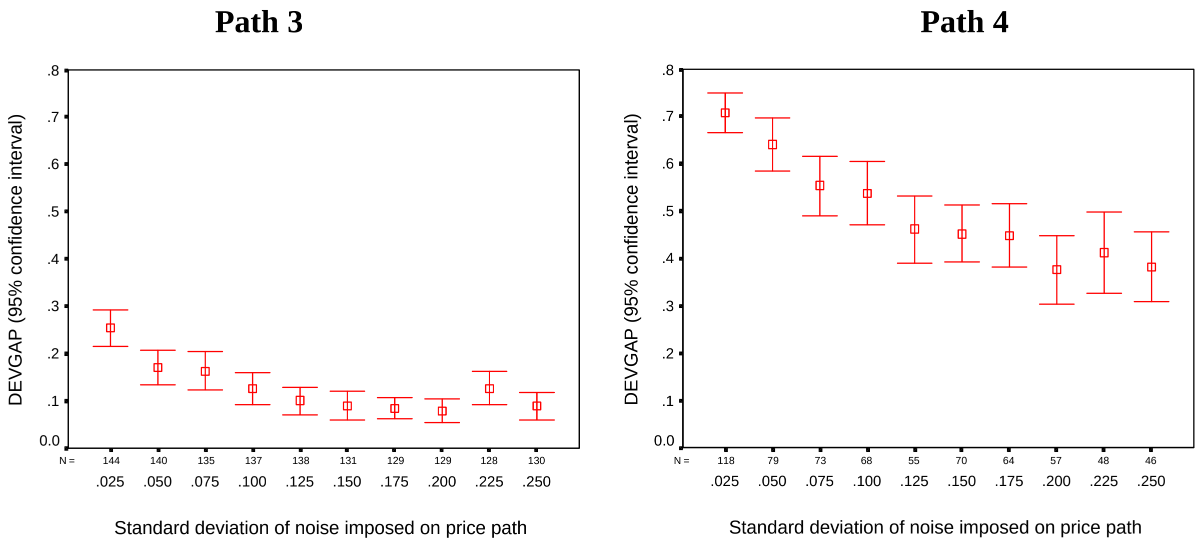


Figure 17. The 95% confidence interval for the net worth gap ratio normalized relative to the no-noise net worth (DEVGAP) at different standard deviations (permanent price change)

To support the above conclusion, we can observe the average labor demand (average over the time periods) for the FAM and the optimal solutions. In Figure 18, the two extreme cases, with standard deviation 0.025 and 0.25, are plotted. At low standard deviations, the distribution of the solutions is nearly vertical, implying that the FAM adjusts when it should not (to achieve optimality). When the variance is high, labor demand deviates substantially from the deterministic case, but the distribution of this deviation approaches the 45° line relative to the low variance deviation.

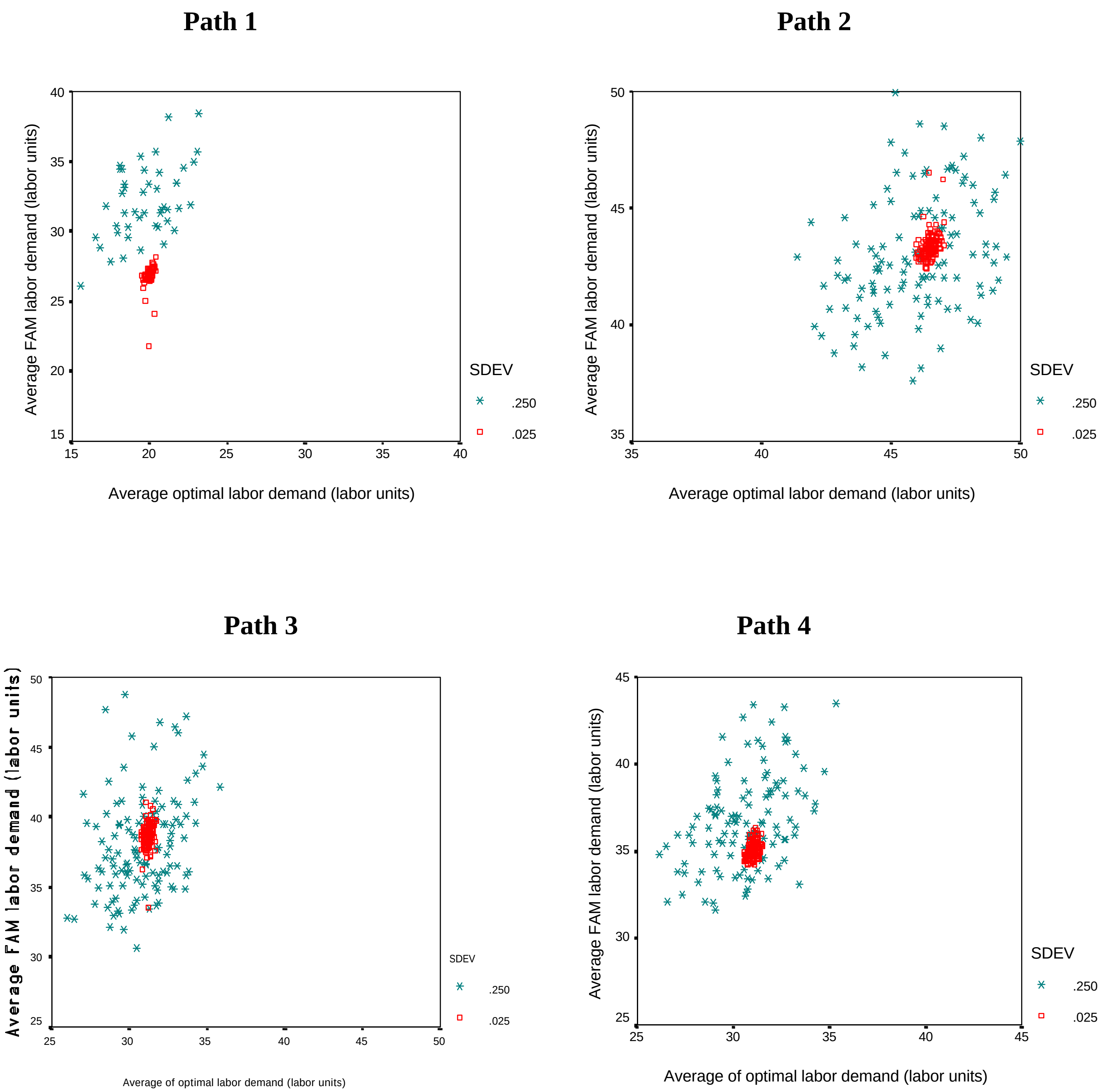


Figure 18. Average labor demand over time: comparing low and high variance results

The variation in labor demand seems to indicate that the similarity between rule-based behavior and optimal behavior in terms of NPV does not carry over into the labor hiring/firing decisions. The reason this result occurs is that objective functions are often quite flat in the neighborhood of optimal solutions, implying that while the NPV changes little even with high noise, the labor demand patterns may deviate substantially, even with relatively low noise levels.

Conclusions

In the context of the literature on bounded rationality, this paper presents a novel empirical approach using fuzzy systems control theory. The main objective is to evaluate the performance of such an approach relative to methods that adopt explicit optimization techniques to describe agent behavior (*as-if-optimizers*). A secondary objective is to develop a framework that lends itself to applied research in situations where standard methods do not perform satisfactorily. In the proposed approach, the “economic agent” uses believably simple rules in coping with complex situations, and these rules can be estimated. The paper uses concepts and methods from fuzzy control theory usually employed in electrical engineering applications.

The approach taken is to generate a sample that would stack the cards in favor of optimization methods by using a sample generated by a perfect foresight, optimal control model. After illustrating that even in such an extreme situation the estimated rules could satisfactorily replicate the underlying optimizing behavior, we proceeded by presenting the interesting features of rule-based behavior in the presence of noise by comparing it to optimal solutions in the same situation. The results indicate sensitivity of the rule-based system at low noise levels (with labor demand deviating from optimal without, however, much impact on NPV). At higher noise levels, there is a further, but less than proportional, deviation from optimal labor demand and it does affect NPV substantially.

The first objective, to show that an appropriately trained rule-based system can behave as if it were an optimizing agent, is accomplished. Having established that there is very close consistency with the data for both the rule-based and the optimizing-agent approaches (with the latter by construction), we consider why the rule-based approach should be the preferred option in a variety of situations. First, we argue that the approach is a "simpler" representation of the firm's operation because it is based on a very simple and straightforward analysis (as put forth by Heiner) concerning the economic motivation for rule adoption. The simple learning process described in this paper is consistent with Alchian's view that behavior should be based on actions that have been tried by at least some economic agents in the past, given that identifying possible action is in itself a process. Furthermore, the method proposed here requires only local knowledge about prices by requiring only a forecast of prices two periods into the future instead of knowledge of the whole price path.[20] The second, and more interesting point of argumentation concerns how "fruitful" the rule-based approach is found to be. The main advantage is that the theory can yield predictions in a wider domain of applications for which the *as if* optimization framework does not apply, such as

1. fluctuating situations in which the agents do not have the time to learn the optimizing behavior (assuming the FAM captures the "real" dynamic behavior);
2. applications for which the data come in qualitative form, for example, from surveys describing the confidence of entrepreneurs in local institutions;
3. linking the approach to game theory to analyze institutions, their rules, and the impact of introducing new rules.

Finally, we believe this approach could be the starting point for further empirical research on the interaction between rule-based agents, the micro-foundations of macroeconomics, the validation of competing models in economics, and the analysis of how rules are generated.

Along with advantages and interesting applications, there are some considerable disadvantages that come with the rule-based approach: (i) all the standard tools of the optimization framework are precluded; including the use of shadow prices, general results concerning markets, and the strong normative prescriptions that have been developed in neoclassical economics; (ii) dropping the hypothesis of explicit optimizing agents requires a substantial data set to estimate behavioral rules.

Given the introductory nature of this paper, it leaves much room for future

[20] Obviously, the optimization problem could be solved repeatedly with a time horizon of two periods, requiring the same amount of knowledge as the FAM ; however, optimal control models are very sensitive to the terminal conditions if solved over such brief time intervals (at reasonable discount rates).

extensions. The estimation procedure for the rules can be improved by applying more advanced methods that are already available in the literature (Gersho and Gray, 1992). The learning process can be represented as occurring over time rather than as a static estimation process. This can be done following the approaches adopted in the “classifier systems” literature using genetic algorithms (Holland and Miller, 1991) and the innovative methods of evolutionary game theory (Marimon, 1993). Another potentially fruitful area is to investigate convergence of a FAM system to equilibrium solutions in more complex applications.

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Appendix: Parameters of Optimal Control Model used for Sample Generation

Initial condition factor demand	$FD0_{"labor",t}$	= 30 units of labor
	$FD0_{"capital"t}$	= 70 units of capital
Initial output	X0	= 100 units
Wages (labor, capital)	$WF_{f,t}$	= 1
discount rate	r	= 0.003 per period

The optimal control model was run using the price paths presented in Figure 6 and with the following set of adjustment costs:

unit hiring/firing costs for labor 0.06 0.11 0.15 0.35 0.5 0.65 0.82 1.0

The costs of hiring and firing are assumed equal.

The sample generated will depend strongly on the discount rate and the factor intensity of the production technology. In the application presented here the technology is capital intensive and this may affect the results, both in terms of the rules that are estimated and the relative performance of the rule-based system relative to optimal.

After the rules were estimated, all the runs were carried out using a hiring/firing unit adjustment cost of 0.15 (for labor). Capital stock was fixed to 70 units in the sample generation, the estimation, and the simulations.

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